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Design of Waveguide Circulators with Chebyshev Characteristics Using Partial-Height Ferrite Resonators

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***Abstract*—This paper outlines a step-by-step approach to the design of waveguide circulators using partial-height resonators, which incorporates every linear dimension of the device. The approach used consists of defining the physical variables of the ferrite region in terms of the frequency, VSWR, and bandwidth specification. It also incorporates the definition of the length and admittance level of the radial transformer. The model employed is essentially a two-mode one, with the third mode separately adjusted to exhibit an ideal electric-wall boundary condition at the terminals of the junction.**

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I. Introduction

ALTHOUGH THE 1-PORT complex gyrator circuit (operating frequency, susceptance slope parameter, loaded Q-factor, gyrator conductance) of waveguide circulators using weakly magnetized quarter-wave-long open ferrite resonators is fairly well understood [1]–[27], there is still no step-by-step procedure for their design in terms of a return loss and bandwidth specification. This is in part due to the fact that the radial impedance transformer used in these devices is a nonuniform line whose dimensions are dependent upon both the gyrator conductance and its terminal plane, and it is in part due to the difficulty in

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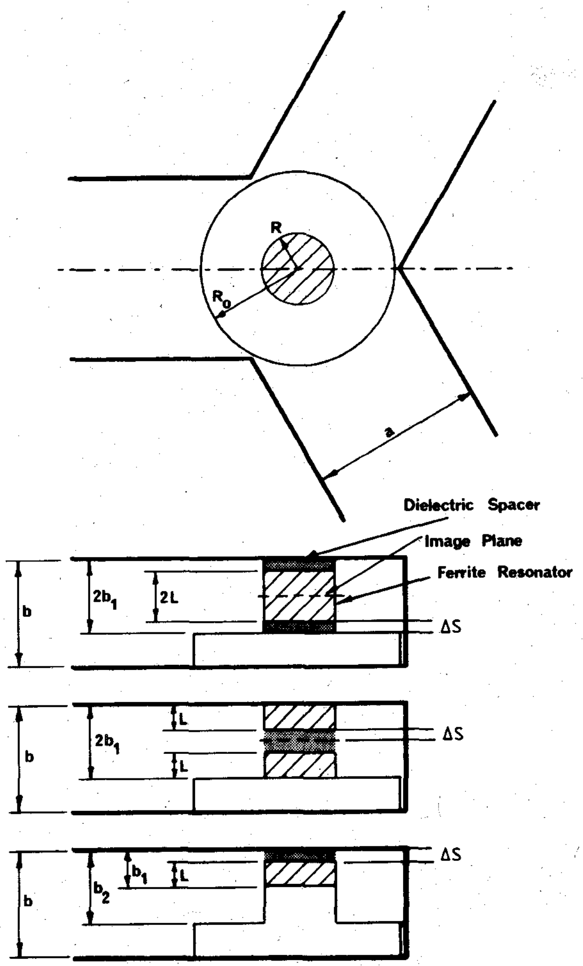

Fig. 1. Schematic diagrams of waveguide circulators using partial-height resonators.

easily describing the boundary between the radial and rectangular waveguides. The purpose of this paper is to remedy this situation. Fig. 1 depicts the three standard commercial circulators using partial-height resonators discussed here. The configurations in Fig. 1(a) and (b) are dual and are described by a single set of variables. The geometries in Fig. 1(a) or (b) and Fig. 1(c) are also identical, except that the susceptance slope parameter of the former ones are half that of the latter one [9]. The degenerate frequencies of the counter-rotating modes in each of these devices are determined by those of an open demagnetized ferrite circular or triangular waveguide open-circuited at one end and short-circuited at the other [2]–[5], [7], [19]. These are then split in the usual way by magnetizing the resonator. The frequency of the in-phase one is determined by a quasi-planar resonance, which is determined by the position of the image or waveguide wall [2], [3], [8], [27].

The design procedure used in this paper to describe this class of circulator is based on a weakly magnetized model of the junction which omits the influence of higher order modes on the gyrator circuit and is determined by the intersection of the upper and lower branches of the first two pairs of split modes in the resonator. It is dependent upon both the radial wavenumber of the ferrite resonator and its magnetic variables. The design starts by calculating the length (L_O) of the resonator from a statement of the operating frequency and radius (R) of the open ferrite waveguide. This condition establishes a magnetic wall at the contiguous terminals of the resonator and fixes the frequencies of the degenerate counter-rotating modes of the device. The boundary condition for the in-phase mode is next met by establishing an electric wall at the terminals of the contiguous wall of the resonator by adjusting the spacing between its open flat face and the image or waveguide wall. The calculation proceeds by determining the correction (ΔL) to the overall length (L_o) of the open ferrite resonator due to the perturbation of its frequency by the image wall. This condition is described in terms of a lumped element resonator in shunt with a distributed circuit representing the counter-rotating resonances [14]. The junction now exhibits a 1-port complex gyrator circuit whose susceptance slope parameter (B') is fixed by the details of the physical varibles employed [9], and whose loaded Q-factor must satisfy the weakly magnetized resonator model adopted in the approximation problem. Network theory is utilized next to investigate the synthesis problem. If the loaded Q-factor is constrained by the lower bound imposed by the weakly magnetized resonator model, then a network solution is not guaranteed and the circuit specification may have to be altered or relaxed, or the radial wavenumber must be modified. Once the network problem is met, the split frequencies of the two counter-rotating modes and therefore the magnetization (M_0) of the ferrite material may be evaluated [26]. The radial terminal (R_0) at which the input admittance is real may now be calculated from a knowledge of the gyrator conductance (G). This definition of the input terminals leads to a modification of that used earlier in terms of the characteristic plane [6]. Finally, the admittance level (Y_t) of the radial transformer is evaluated in terms of the radial length and gyrator level of the junction. This, in turn, fixes the height (b_1) of the radial waveguide for the geometries in Fig. 1(a) and (b) and b_2 for that in Fig. 1(c). In the case of the latter configuration, the design is now complete since b_2 is an independent variable. In the former two cases, b_1 is not an independent parameter since its value must be consistent with that obtained earlier in satisfying the phase angles of the three eigennetworks. If the two values for b_1 obtained from the two design statements are not consistent, a new value for R is chosen and the procedure is repeated until they are compatible. However, this loop may also be avoided by making b_2 not equal to $2b_1$.

The paper includes an evaluation of one high-quality circulator with Chebyshev characteristics in WR284 waveguide which is in good agreement with theory.

II. Circulation Frequency of Waveguide Circulators Using Partial-Height Resonators

The construction of conventional junction circulators involves the adjustment of two counter-rotating split eigennetworks and one in-phase one. The counter-rotating eigennetworks employed in the class of circulators discussed here consist of either a quarter-wave-long magnetized ferrite waveguide with ideal magnetic walls short-circuited at one end and open-circuited (or terminated by an image wall) at the other end, or a similar half-wave-long waveguide with ideal magnetic walls open-circuited (or terminated by image walls) at both ends [2], [3]. Since the two versions are dual, a quarter-wave-long prototype is adequate to describe both situations. The in-phase eigennetwork is a quasi-planar circuit and is common for the two configurations also [27]. Although the operating frequency of the device is mainly determined by the demagnetized counter-rotating networks, all three must be commensurate in order for the device to exhibit a useful 1-port complex gyrator circuit. The characteristic equation for the demagnetized resonator is formed by representing the junction by a dielectric waveguide with ideal magnetic walls terminated at one end by a short-circuit boundary condition, and at the other by a contiguous waveguide terminated by the image wall [5]–[7]

$$\zeta_0\left[\frac{\epsilon_f k_0}{\beta_0}\cot(\beta_0 L)-\frac{\epsilon_d k_0}{\alpha}\coth(\alpha\Delta S)\right]=0. \qquad (1)$$

This equation may be employed to calculate the dimensions of the junction once the boundary condition of the in-phase mode is formulated.

The quantities in the preceding equation are defined in the usual way by

$$\beta_0^2=\left(\frac{2\pi}{\lambda_g}\right)^2=\left(\frac{2\pi\sqrt{\mu_d\epsilon_f}}{\lambda_0}\right)^2-\left(\frac{1.84}{R}\right)^2 \qquad (2)$$

$$\alpha^2=\left(\frac{1.84}{R}\right)^2-\left(\frac{2\pi\sqrt{\epsilon_d}}{\lambda_0}\right)^2 \qquad (3)$$

$$k_0=\left(\frac{2\pi}{\lambda_0}\right) \qquad (4)$$

$$\zeta_0=\sqrt{\frac{\epsilon_0}{\mu_0}} \qquad (5)$$

$$\mu_d=\frac{1}{3}+\frac{2}{3}\left[1-\left(\frac{\gamma M_0}{\mu_0\omega}\right)^2\right]^{1/2}. \qquad (6)$$

ϵ_d is the relative dielectric constant of the region between the open dielectric resonator and the image or waveguide wall, ϵ_f is that of the ferrite material, μ_d is the demagnetized permeability of the ferrite resonator, R, L, and ΔS are defined in Fig. 1, and the other quantities have the usual meaning.

The influence of the image wall on the frequency of the open resonator may be understood by writing L in (1) in terms of the length L_o of the open resonator

$$L=L_o-\Delta L. \qquad (7)$$

The result, in the vicinity of $\beta_0 L_o=\pi/2$, is

$$\zeta_0\left[\frac{\epsilon_f k_0}{\beta_0}\cot(\beta_0 L_o)+\frac{\epsilon_f k_0}{\beta_0}\tan(\beta_0\Delta L)-\frac{\epsilon_d k_0}{\alpha}\coth(\alpha\Delta S)\right]=0. \qquad (8)$$

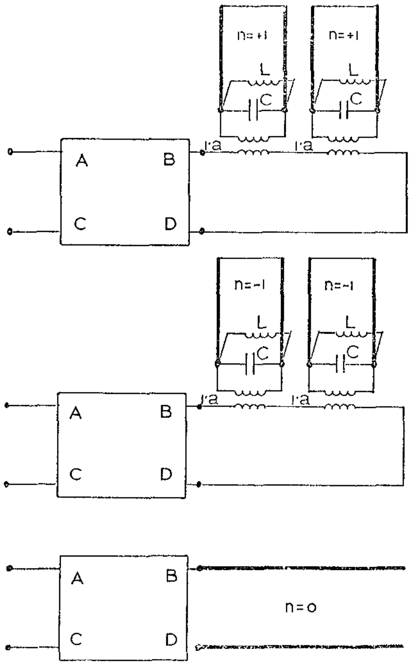

Fig. 2. Eigennetworks of waveguide circulators using coupled quarter-wave-long resonators or single half-wave-long resonators.

Using this representation, the perturbation of the open dielectric resonator due to the image or waveguide wall is described by a lumped element resonator

$$\frac{\epsilon_f}{\beta_0}\tan(\beta_0\Delta L)-\frac{\epsilon_d}{\alpha}\coth(\alpha\Delta S)=0 \qquad (9)$$

and the open resonator is realized by a quarter-wave-long short-circuited open dielectric waveguide

$$\cot(\beta_0 L_o)=0. \qquad (10)$$

Figs. 2 and 3 depict the eigennetworks for the counter-rotating modes obtained by replacing β_0 by $\beta_\pm$ [26]. They also include the quasi-planar eigennetwork for completeness.

A. *Adjustment of Quasi-Planar In-Phase Eigennetwork*

Although in the 1-port approximation of the circulator the frequency variation of the in-phase eigennetwork is neglected compared to that of the counter-rotating ones, it is still necessary to establish the proper phase angle for this network. This in-phase quasi-planar network determines the spacing between the open resonator and image wall. For an ideal circulator, the reflection coefficients s_0 for the in-phase mode at $r=R$ and R_0 are

$$s_0=-1, \quad r=R \qquad (11)$$

$$s_0=+1, \quad r=R_0. \qquad (12)$$

Since for a radial junction these two conditions cannot be satisfied simultaneously with the value of R_0 obtained from the boundary condition of the counter-rotating modes, they have in the past placed upper and lower bounds on the filling factor (k) of the quasi-planar resonator [6]. However, experience indicates that the former condition is

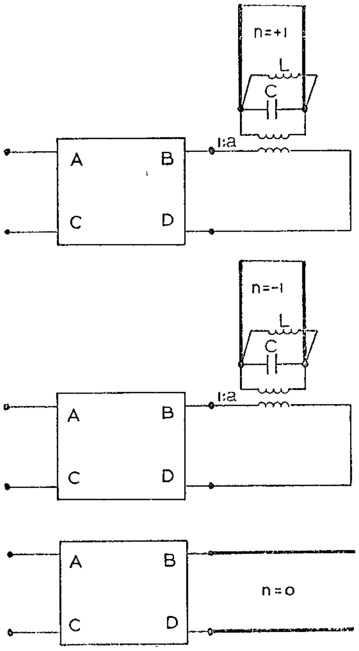

Fig. 3. Eigennetworks of waveguide circulators using single quarter-wave-long resonator.

to be preferred, so that the boundary condition $s_0 = -1$ will be adopted here [8].[1]

Taking $s_0 = -1$ at $r = R$ gives

$$J_0\left(k_0\sqrt{\epsilon_{\text{eff}}}\,R\right) = 0 \tag{13}$$

or

$$k_0\sqrt{\epsilon_{\text{eff}}}\,R = 2.405. \tag{14}$$

ϵ_{eff} is the effective dielectric constant of the composite ferrite-dielectric region

$$\epsilon_{\text{eff}} = \frac{\epsilon_f \epsilon_d}{\left[(1-k)\epsilon_f + k\epsilon_d\right]}. \tag{15}$$

The filling factor is related to the physical variables of the junction by

$$k = \frac{(L_o - \Delta L)}{(L_o - \Delta L) + \Delta S}. \tag{16}$$

It is now necessary to simultaneously satisfy (1) and (13) for ΔS (or ΔL) for various values of $k_0 R$. However, not only is (13) incompatible with practice, but as Fig. 4 indicates, there is no solution to this boundary condition as it stands either without making some assumptions about the location of the effective electric and magnetic walls of the resonator for each mode. This difficulty may be catered for by displacing the lateral wall of the resonator for both the counter-rotating and in-phase modes outwardly from R to R_{eff}. Since ϵ_{eff} is much lower than ϵ_f, the lateral displacement for the in-phase mode is the more substantial. Some recent careful experimental work indicates that a suitable polynomial approximation for the filling factor which is applicable to either single and coupled resonator junctions with $\epsilon_d = 1$ is [27]:

$$k \approx 0.2196 + 2.204(k_0 R) - 1.785(k_0 R)^2$$
$$\epsilon_f = 15, \quad 0.75 < k_0 R < 0.95 \tag{17}$$

or

$$k_{\text{eff}} R \approx 0.6748 + 2.9827(k_0 R) - 2.093(k_0 R)^2$$
$$\epsilon_f = 15, \quad 0.75 < k_0 R < 0.95. \tag{18}$$

Scrutiny of the graphical solution in Fig. 4 indicates that an accurate description of $k_0 R$ is also required for the counter-rotating modes if L and k are to be meaningful. This suggests that the idealization of the lateral walls of the resonator by magnetic walls leaves something to be desired. Thus, the full theory must be used [23], [25], or $k_0 R$ must be reset on the basis of the data in [7]. Likewise, the use of (6) to describe the permeability of the magnetized ferrite is not strictly correct and leads to some error in the calculation of L which must be experimentally catered for.

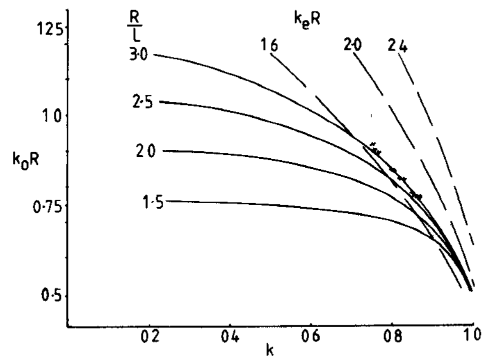

Fig. 4. Graphical solution of turnstile junction.

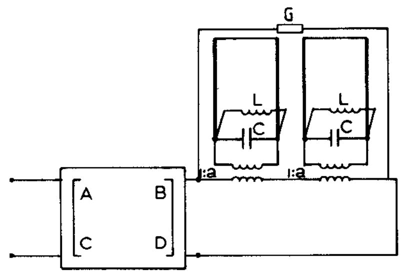

Fig. 5. 1-port equivalent circuit of waveguide circulator using coupled quarter-wave-long resonators or single half-wave-long resonators.

B. Gyrator Conductance and Loaded Q-Factor of Circulators Using Weakly Magnetized Partial-Height Resonators

Once the in-phase and counter-rotating modes of a junction have been correctly adjusted, its eigennetwork model may be replaced by a 1-port equivalent circuit. The 1-port complex gyrator circuits for the two devices considered here are illustrated in Figs. 5 and 6. For circulators using weakly magnetized resonators, these 1-port networks exhibit gyrator conductances described by the following well-known relationship:

$$g = \sqrt{3}\, b'\left(\frac{\omega_+ - \omega_-}{\omega_0}\right). \tag{19}$$

[1] This is a somewhat different view from that adopted in [6].

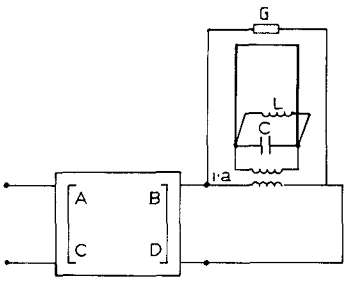

Fig. 6. 1-port equivalent circuit of waveguide circulator using single quarter-wave-long resonator.

$\omega_\pm$ are the resonant frequencies of the magnetized open resonator, ω_0 is that of the demagnetized one, g is the normalized gyrator conductance, and b' is the normalized susceptance slope parameter. Any two of these three quantities are sufficient to describe the gyrator equation.

The difference between the split frequencies of the magnetized open ferrite resonator may be determined by omitting the image wall term in (1) and replacing β_0 by $\beta_\pm$ as follows:

$$\cot \beta_\pm L_o = 0. \tag{20}$$

$\beta_\pm$ are the split phase constants of the magnetized open ferrite resonator, and L_o is the length of the demagnetized open ferrite resonator.

$\beta_\pm$ may be exactly evaluated using duality between a ferrite-filled circular waveguide with an ideal electric-wall boundary condition and one having an ideal magnetic wall. The result is dependent upon both $k_0 R$ and κ/μ. For $k_0 R = 0.80$, it is given by [26]

$$\left(\frac{\omega_+ - \omega_-}{\omega_0}\right) \approx 0.621\left(\frac{\kappa}{\mu}\right) - 0.061\left(\frac{\kappa}{\mu}\right)^2 + 0.066\left(\frac{\kappa}{\mu}\right)^3, \quad 0 \leqslant \frac{\kappa}{\mu} \leqslant 0.5. \tag{21}$$

Fig. 7 gives one experimental plot of the split frequencies of a magnetized disk resonator.

Equation (19) is also sometimes written as

$$\frac{1}{Q_L} = \sqrt{3}\left(\frac{\omega_+ - \omega_-}{\omega_0}\right). \tag{22}$$

The weakly magnetized resonator model employed here disregards the influence of higher order modes on the description of the gyrator circuit. The intersection of the upper and lower branches of the first two pairs of split modes in this resonator is in fact dependent upon both $k_0 R$ and κ/μ [26]. Fig. 8 summarizes this result. This data suggests that the lower bound on Q_L (or the upper bound on κ/μ) may be adjusted by varying $k_0 R$.

The complex gyrator admittance may also be written in terms of g and b' in (19) as

$$y = g - j2\delta b' \tag{23}$$

where 2δ is the normalized frequency variable.

Once the gyrator level is set by the synthesis problem, it is possible to evaluate the electrical length of the nonuniform radial transformer.

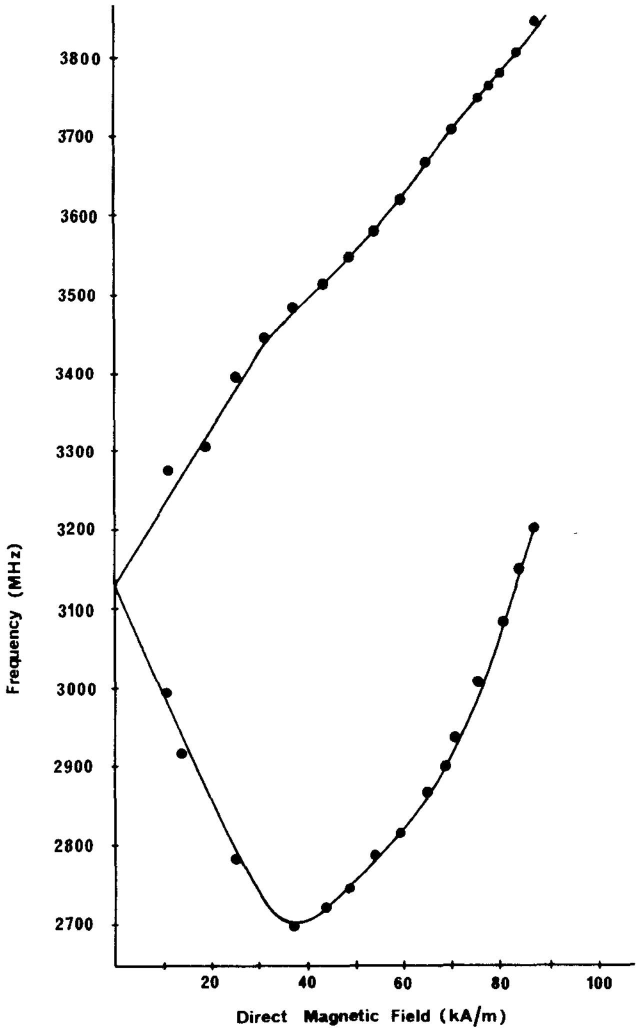

Fig. 7. Split frequencies of loosely coupled quarter-wave-long resonator versus direct magnetic field.

C. Susceptance Slope Parameter of Circulators using Partial-Height Ferrite Resonators

The susceptance slope parameter of the 1-port gyrator circuit may be determined by forming the total transverse admittance at the input terminals of the demagnetized resonator in Fig. 1(c) [9], [13], assuming that TM conditions prevail in the dielectric waveguide:

$$\zeta_1 = -j\zeta_0\left[\frac{\epsilon_f k_0}{\beta}\cot(\beta L_o) + \frac{\epsilon_f k_0}{\beta}\tan(\beta \Delta L) - \frac{k_0}{\alpha}\coth(\alpha \Delta S)\right]. \tag{24}$$

This admittance may be realized as a distributes network in shunt with a lumped element resonator in the manner indicated in Figs. 2 and 3.

The admittance of the eigennetwork is now assumed to have the form below, provided the characteristic impedance of the rectangular waveguide is taken as in (29) below:

$$y_1 = n^2 \zeta_1. \tag{25}$$

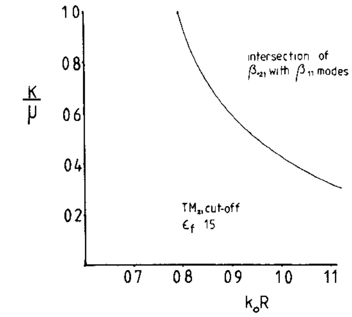

Fig. 8. Intersection between upper and lower branches of first two split modes in open ferrite resonator.

n^2 represents an ideal transformer which described the change in admittance between the eigenresonator and eigennetwork at the terminals of the junction. One empirical relationship that appears to satisfy the experimental work with $\epsilon_d = 1$ is [9]

$$n^2 \approx \frac{1}{3}\frac{ab}{(\pi R^2)}. \tag{26}$$

ab is the cross-sectional area of the rectangular waveguide, and πR^2 is that of the open demagnetized ferrite waveguide.

The susceptance slope parameter B' of the open demagnetized ferrite resonator can be evaluated in terms of the physical variables of the eigennetwork by making use of its definition below:

$$B' = \frac{\omega_0}{2}\frac{\delta y_1}{\delta\omega}\Big|_{\omega_0}. \tag{27}$$

ω_0 is obtained by satisfying (1) or (24) with $\zeta_1 = 0$ [5]–[7]. Evaluating the preceding equation leads to the required result [6], [9]:

$$B' = n^2\zeta_0\left\{\frac{\pi\epsilon_f^2}{4}\left(\frac{k_0}{\beta_0}\right)^3 + \frac{\epsilon_f^2}{2}\left(\frac{k_0}{\beta_0}\right)^3 \cdot\left[\tan(\beta_0\Delta L) - \frac{\beta_0\Delta L}{\cos^2(\beta_0\Delta L)}\right] + \frac{1}{2}\left(\frac{k_0}{\alpha}\right)^3\left[\coth(\alpha S) + \frac{\alpha S}{\sinh^2(\alpha S)}\right]\right\}. \tag{28}$$

Normalizing B' to the impedance of the rectangular waveguide with TE propagation and wavelength λ_{gw} yields the required result

$$b' = \frac{B'}{Y_0} = \frac{1}{\zeta_0}\left(\frac{\lambda_{gw}}{\lambda_0}\right)B'. \tag{29}$$

It is experimentally observed that the susceptance slope parameters for the coupled disks and single cylinder geometries are half that given by (28) which applies to a single disk resonator [6]. Fig. 9 indicates the normalized susceptance slope parameter of such a junction [9].[2]

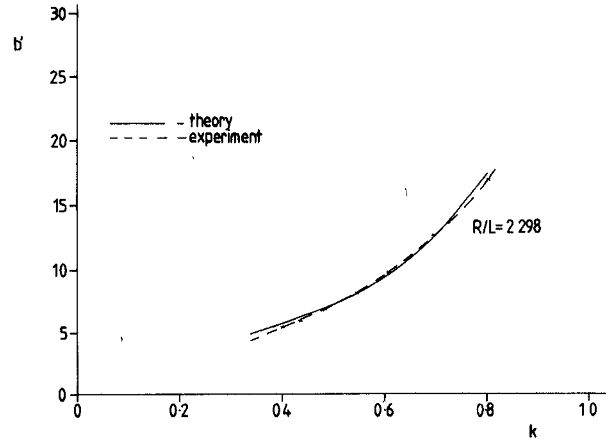

Fig. 9. Normalized susceptance slope parameter of single quarter-wavelong resonator [9].

Equation (28) is readily modified to cater for resonators employing triangular cross sections in the manner outlined in [26].

III. Synthesis of Waveguide Circulators Using Partial-Height Resonators

In the synthesis problem, the variables of the complex gyrator circuit (g, b', Q_L) are usually determined by the maximum and minimum values of the VSWR, S(max) and S(min) and the bandwidth W of the circulator specification. This problem is fully discussed in [11], [12] and will not be repeated here. However, it is recalled that varying S(min) has a significant influence on the absolute values of g and b'.

In the synthesis of circulators using partial-height resonators, the susceptance slope parameter and the lower bound on the loaded Q-factor are fixed by the choice of the radial wavenumber of the resonator. If the lower bound for Q_L is adopted for design, then the gyrator conductance is also fixed. The latter quantities as well as S(max) may then be taken as the independent variables and S(min) and the normalized bandwidth W may be taken as the dependent ones and the ensuing network problem may be tested for realizability. If no solution is possible, or if S(min) or W are unacceptable, then b' or Q_L must be modified by varying either the radial wavenumber of the resonator or its shape.

Fig. 10 depicts the relationship between the theoretical return loss (with $S(\text{min}) = 1$ and $Q_L = 2$) and the bandwidth W achieved here as well as some experimental results on circulators using single cylinders and coupled disks resonators obtained by a number of industrial organizations. This result suggests that the weakly magnetized theory developed in this paper is suitable for the realization of devices with 30-percent bandwidths at the 20-dB return-loss frequencies.

[2]The factor 2/3 normally associated with [6, eq. (24)] and [9, eq. (28)] has been absorbed in the turns ratio in (26).

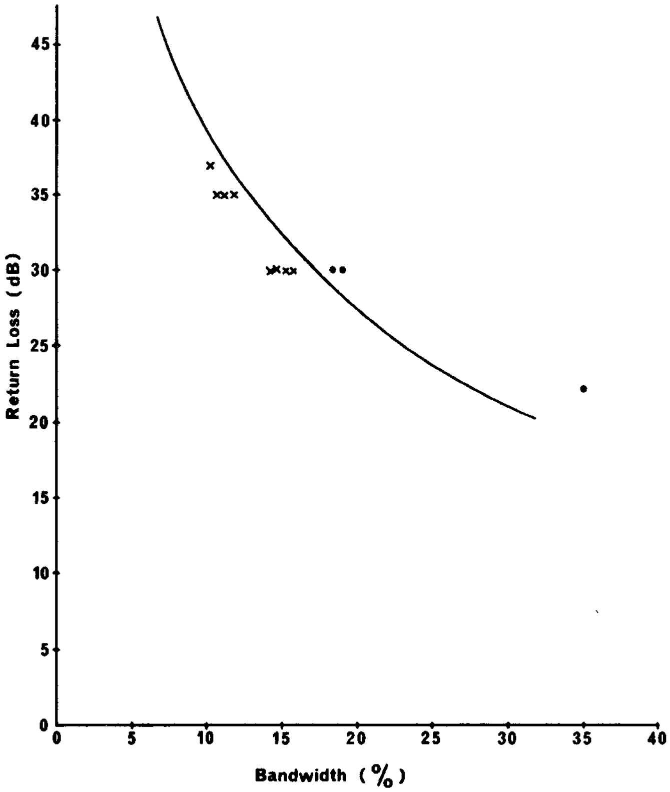

Fig. 10. Relationship between return-loss and bandwidth (with $S(\min)=1$). (Experimental data courtesy Ferranti Ltd., Thompson-CSF, Microwave Associates, Marconi.)

A. Overall Frequency Response of Circulator

The overall frequency response of the circulator can be constructed once the physical variables of the junction are fixed. The admittance at the input terminals is now formed in terms of the radial waveguide $ABCD$ matrix and the complex gyrator admittance. The result is:

$$Y_{\text{in}} = m^2 \cdot \frac{jC + DY_L}{A + jBY_L}. \tag{30}$$

For the radial waveguide, the $ABCD$ matrix has the following standard form:

$$A = \frac{\pi(k_0 R)}{2}\left[J_n(k_0R_0)Y_n'(k_0R) - J_n'(k_0R)Y_n(k_0R_0)\right] \tag{31}$$

$$B = \frac{\pi(k_0 R)}{2\zeta_0}\left[J_n(k_0R)Y_n(k_0R_0) - J_n(k_0R_0)Y_n(k_0R)\right] \tag{32}$$

$$C = \zeta_0\frac{\pi(k_0 R)}{2}\left[J_n'(k_0R)Y_n'(k_0R_0) - J_n'(k_0R_0)Y_n'(k_0R)\right] \tag{33}$$

$$D = \frac{\pi(k_0 R)}{2}\left[J_n(k_0R)Y_n'(k_0R_0) - J_n'(k_0R_0)Y_n(k_0R)\right] \tag{34}$$

with $n=1$.

m^2 is the turns ratio of an ideal transformer which represents the transition between the rectangular and radial waveguides. It is calculated assuming that the transformer terminals coincide with those of the radial waveguide formed by the junction of the rectangular waveguides. Although this is not quite the case, the two are sufficiently close to permit the region between the terminals of the circulator and the radial cavity to be neglected. Taking a 9-GHz circulator in WR90 waveguide as an example indicates that $k_0R_0 = 2.48$ at the terminals of the radial cavity, whereas it is typically 2.36 in practice.

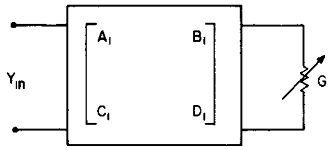

Fig. 11. Definition of input terminals of radial transformer terminated by variable conductance.

Applying the boundary conditions at the terminals of the radial and rectangular waveguides gives the turns ratio between the radial and rectangular waveguides as

$$m^2 \approx \frac{3\sin(n\phi)}{2n\pi\phi}\left[\frac{\sin(3/2-n)\phi}{(3/2-n)} + \frac{\sin(3/2+n)\phi}{(3/2+n)}\right]. \tag{35}$$

This equation is evaluated with $n=1$ and

$$\phi = \pi/3. \tag{36}$$

The equivalent circuits which apply here are depicted in Figs. 5 and 6.

B. Input Terminals of Radial Waveguide Circulator

For a magnetized radial waveguide junction, the notion of characteristic plane used in [6] must be modified to account for the fact that the output terminals of the line are no longer an open circuit but a variable conductance (Fig. 11) determined by the gyrator level of the junction. The input terminals of such a junction now coincide with those at which the input admittance is real. Since the radial line is nonuniform, this plane is now a function of the gyrator level.

The input admittance of the circulator is given by

$$Y_{\text{in}} = m^2\frac{jC + DG}{A + jBG} \tag{37}$$

where it is assumed that the gyrator admittance is real at the center frequency.

Rearranging Y_{in} leads to

$$Y_{\text{in}} = m^2 \cdot \frac{(AD+BC)G + j(AC - BDG^2)}{A^2 + B^2G^2}. \tag{38}$$

Setting the imaginary part to zero gives the input terminals of the device as

$$AC - BDG^2 = 0 \tag{39}$$

whereas the real part is determined by

$$m^2\frac{(AD+BC)G}{A^2+B^2G^2}. \tag{40}$$

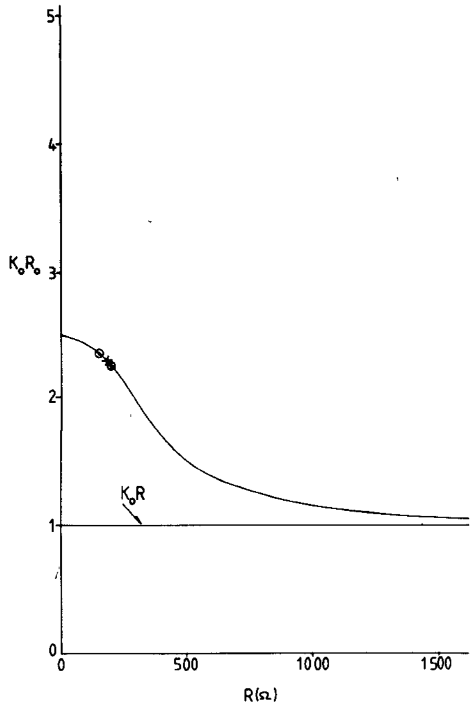

Fig. 12. $k_0 R_0$ versus gyrator resistance for $k_0 R = 1$.

Fig. 12 shows the radial terminals of the junction as a function of the gyrator level for $k_0 R = 1.0$.

IV. Radial Transformer Admittance

Once the gyrator level and radial length of the radial transformer are known, the admittance level of the transformer can be evaluated using (40), which gives the real part of the input admittance of the circulator

$$\zeta_0\left(\frac{\lambda_0}{\lambda_g}\right) = m^2 \frac{(AD+BC)G}{A^2 + B^2G^2}. \tag{41}$$

If $G \geqslant 3\ \zeta_0(\lambda_0/\lambda_g)$ and ζ_0 in B and C is replaced by $\zeta_0 b/b_2$, the following approximation applies:

$$\zeta_0\left(\frac{\lambda_0}{\lambda_g}\right) \simeq \left(\frac{b}{b_2}\right)^2 \frac{m^2C}{BG}. \tag{42}$$

This relationship assumes that the impedance levels in the rectangular and radial waveguides are proportional to the narrow dimensions of the waveguides. Equation (42) defines the admittance of the radial transformer Y_T as

$$Y_T^2 = \left(\frac{b}{b_2}\right)^2 \frac{m^2C}{B}. \tag{43}$$

Normalizing G to $\zeta_0(\lambda_0/\lambda_g)$ and taking the VSWR at the center frequency as $S(\max) = r$ leads to

$$\zeta_0\left(\frac{\lambda_0}{\lambda_g}\right) \simeq \left(\frac{b}{b_2}\right)\sqrt{\frac{m^2C}{rgB}} \tag{44}$$

where g is now the normalized gyrator conductance.

This equation may be solved for the waveguide height of the transformer section.

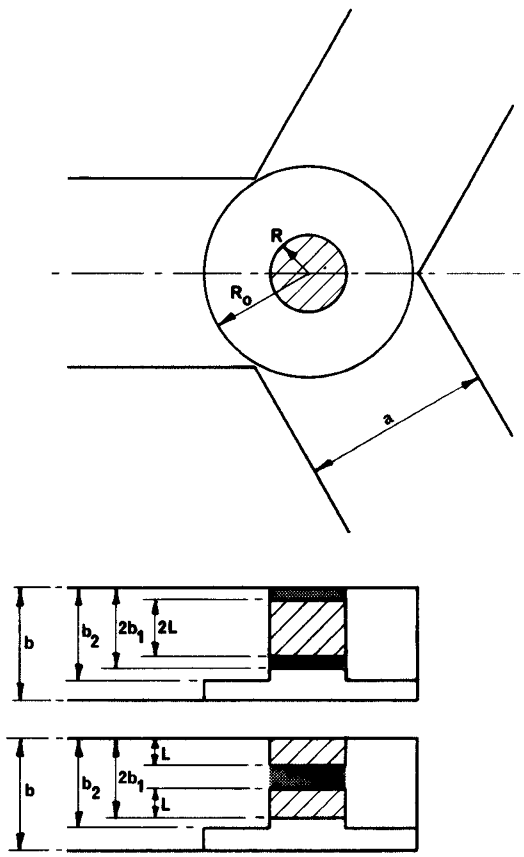

Fig. 13. Schematic diagrams of waveguide circulators using partial-height resonators with $b_2 \neq 2b_1$.

V. Computation

The theory developed in this paper to describe the three important circulator geometries depicted in Fig. 1 has been incorporated into two separate computer programs. One program caters to the coupled disk arrangement or its dual the single cylinder, whereas the second applies to the more simple single disk geometry. The programs take frequency, waveguide size, magnetization (upper bound), and dielectric constant of the garnet or ferrite material, as inputs, and tabulates $S(\max)$ with $S(\min) = 1$ versus bandwidth and the linear dimensions of the junction as outputs.

For the first two geometries, a trial value for the ferrite radius is taken which leads to a junction assembly that will in general be incompatible with the waveguide size so that the computer program incorporates a loop to step the radius until the overall assembly is compatible with the waveguide dimensions. This construction therefore leads to a unique relationship between bandwidth and $S(\max)$ and may well be the explanation why most commercial waveguide circulators exhibit similar characteristics. This difficulty does not arise in the case of the single disk geometry since the transformer impedance is an independent variable in this situation. A computer program is therefore not essential for the design of this circulator in that the calculations involved here are relatively straightforward. The loop in the former cases may also be avoided by making b_2 not equal to $2b_1$ in the manner indicated in Fig. 13.

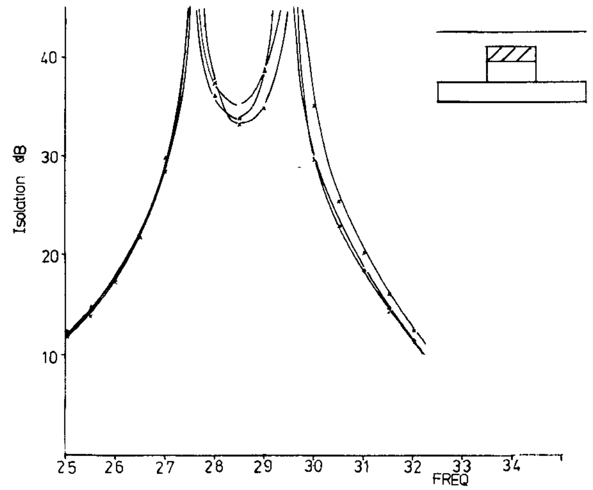

Fig. 14. Experimental frequency response of quarter-wave coupled waveguide circulator in WR284 using single quarter-wave-long resonator.

The step-by-step design of a 2.85-GHz circulator using a single disk junction in WR284 waveguide will now be illustrated. It starts by choosing a trial value for $k_0R(0.89)$ and proceeds to evaluate the length of the resonator (L) and the location of the image wall (ΔS) as well as the radius of the resonator (R). Once the junction configuration is fixed, its susceptance slope parameter and the lower bound on the loaded Q-factor may be evaluated. The results are $b'=11.60$ and $Q_L=1.63$. This calculation also, incidentally, fixes the lower bound for the magnetization of the ferrite resonator. Using these gyrator parameters in conjunction with a return-loss specification of $S(\min)=1$, $S(\max)=1.03$, gives the gyrator level as $g=7.26$ and the required Q as 1.9. Since this value of Q-factor exceeds the lower bound estimate, the calculation may proceed; otherwise a different radial wavenumber must be selected or the specification must be modified. The corresponding bandwidth is now given from the network problem by $W=12.0$ percent. The radius R_0 is next calculated in terms of the gyrator level. The result is $k_0R_0=2.36$ for which $R_0=40.5$ mm. Finally, (44) gives $b_2/b=0.51$, which completes the design procedure.

The calculated and experimental quantities are listed below:

$$k_0R = 0.89 \qquad (0.89)$$
$$k_0R = 2.36 \qquad (2.42)$$
$$R/L = 2.13 \qquad (2.10)$$
$$k = 0.76 \qquad (0.70)$$
$$b_2/b = 0.51 \qquad (0.63)$$
$$\epsilon_d = 1 \qquad (1)$$
$$M_0 = 0.0618\ \text{T} \qquad (0.0680\ \text{T}).$$

The overall frequency response of the device obtained by fine tuning each port is depicted in Fig. 13.

The agreement between theory and practice is good for the first four quantities (which define the phase angles of the eigennetworks) but is less good for the quantity b_2/b (which determines the impedance level of the eigennetworks). This lack or correlation is partly in keeping with the fact that the exact direct field of the experimental junction was not established as a preamble to quarter-wave coupling it, so that some experimental tradeoff between the gyrator conductance and the impedance transformer was necessary.

VI. Conclusions

This paper has developed the synthesis of waveguide circulators with Chebyshev characteristics using open ferrite resonators. The solution of the coupled disk arrangement, or its dual, the single cylinder one, is not possible without recourse to a computer program because of the iteration involved. However, for the single disk geometry, no such iteration is necessary and a simple desk computer is all that is required for design.

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Cyclotron Maser and Peniotron-Like Instabilities in a Whispering Gallery Mode Gyrotron

PETER VITELLO

***Abstract*—The efficiency of the *m*th harmonic electron cyclotron maser interaction for a TE_{mn1} gyrotron oscillator is compared with the $(m-1)$th harmonic peniotron-like interaction. Identical cavities and electron beams are used. Start oscillation conditions from weak-field linear theory are given, as well as optimized nonlinear efficiencies. The peniotron-like interaction leads to optimized efficiencies of ≤ 65 percent, while those for the electron cyclotron maser interaction are limited to ≤ 25 percent in the cases studied.**

THE ELECTRON CYCLOTRON maser interaction in gyrotron devices provides an extremely efficient mechanism for generating high-power microwave radiation [1]–[6]. In the cyclotron maser interaction, electromagnetic (RF) waves in a cavity or waveguide azimuthally bunch an electron beam in which the individual electrons move along helical orbits in the presence of an applied magnetic field. Numerous reviews and theoretical treatments of this instability have already appeared in the literature [1]–[5], [7]–[12]. While the cyclotron maser interaction has been shown capable of generating high microwave power, it is not the sole interaction which takes place in a gyrotron, nor is it necessarily the most efficient.

In this paper, we compare the cyclotron maser interaction with the less well-known [7], [12]–[15] peniotron-like interaction, for a whispering gallery mode TE_{mn1} gyrotron oscillator cavity. Emphasis will be placed on large values of m, and n equal to 1 or 2. The system considered consists of an axis-encircling electron beam (initially centered on the axis) in a right cylindrical cavity of length L and radius R, with a circularly polarized RF standing wave of amplitude E_O and a constant axial guide magnetic field B_O. The initial beam velocities, normalized to the speed of light c, parallel to and perpendicular to the axis are $\beta_{\parallel O}$ and $\beta_{\perp O}$. We will use dimensionless units, with the cavity radius as our scaling parameter. In these units, length is measured in

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